Boundaries in Loop Quantum Gravity

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Introduction and Motivation

The boundary is part of the system

- The simplest Dirac observables are the charge integrals at infinity
  - Linear Momentum: $E_\xi = \frac{1}{8\pi G} \lim_{\rho\to\infty} \oint_{S^2_\rho} d^2\Omega\, \rho^3 E_{ab} n^b \xi^a$
  - Angular Momentum: $J_\omega = \frac{1}{16\pi G} \lim_{\rho\to\infty} \oint_{S^2_\rho} d^2\Omega\, \rho^4 B^{[a}{}_c \hat{\rho}^{b]} n^c \omega_{ab}$

- The first law links charges at infinity to the BH area at finite distance

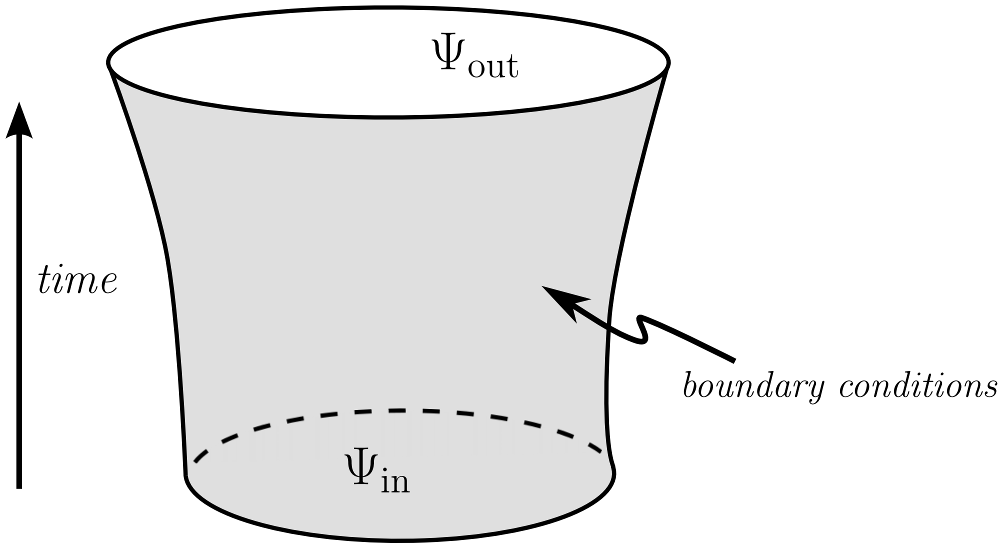
$$\underbrace{\delta M + \Omega\, \delta J}_{\text{at infinity}} = \frac{\kappa}{8\pi G} \delta A$$

➡ The black hole exterior is a Hamiltonian system in a box!

- Boundary at infinity: *asymptotic charges*
- Black hole horizon: *area, surface gravity, edge modes*

Quantum gravity in a box

- If we quantise a field theory in a box, we have to specify boundary conditions.



- Problematic in GR: location of the boundary is itself physical/determined by dynamical variables.

- Potential solution: assign amplitudes to *finite regions* in spacetime [Ashtekar, Rovelli, Barrett]. $\rightsquigarrow$ boundary conditions turn into constraints on the **boundary Hilbert space**.

$$\mathscr{H}_{\mathrm{phys}} = \big(\mathscr{H}_{\partial\Sigma} \otimes \mathscr{H}_{\Sigma}\big)/_{\mathrm{gauge}}$$

[Ashtekar, Beetle, Krasnov, Lewandowski, Thiemann, Sahlmann, Bodendorfer, Oeckl, Rovelli, Freidel, Pranzetti, Donnelly, Marolf, Perez, Speziale, Girelli, Geiller, Dittrich, Goeller, ww,…]

… let us first

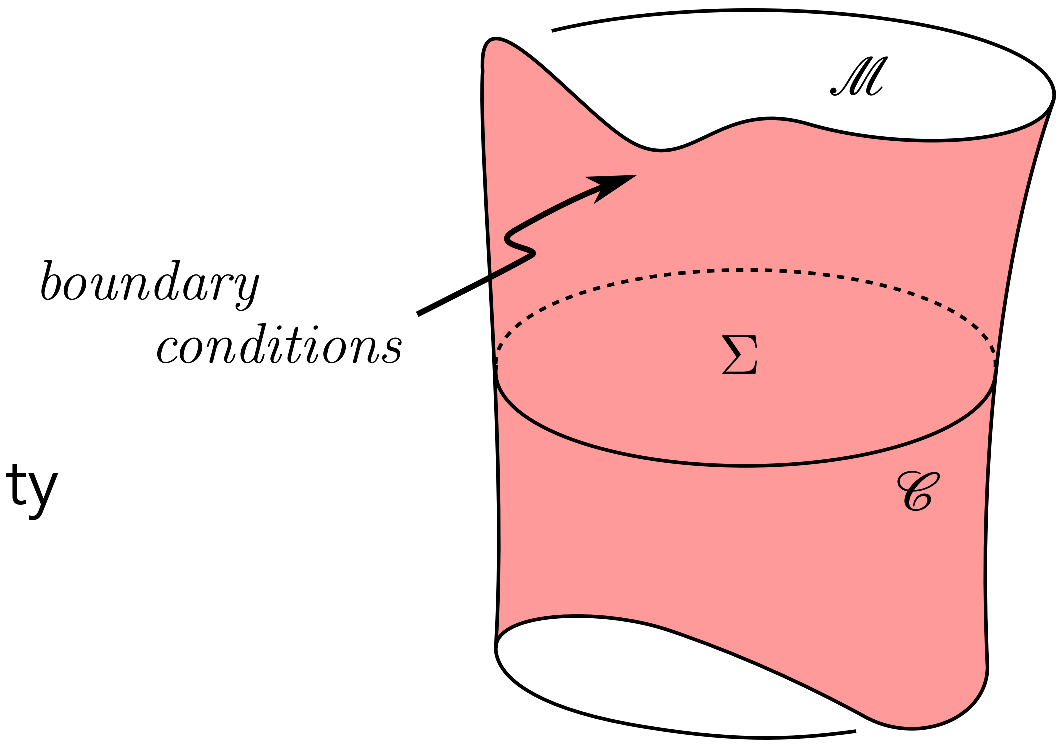
understand the problem in 3d ..

LQG boundary modes in three-dimensional gravity

Quasi-local quantisation of 3d gravity

- Setup: three-dimensional euclidean gravity, vanishing cosmological constant.

- Quasi-local approach: Quantise gravity in a box (finite cylinder).



- Which box? Shape of the box is itself dynamical: depends on the boundary conditions/boundary dynamics.

Boundary CFT in spin network representation: [Dittrich, Geiller, Goeller, Riello, Bonzom, Livine, Perez, Pranzetti, Freidel]

Conformal boundary conditions

Different boxes ~ different boundary conditions ~ different dual boundary field theories.

- *To compare the boundary field theory with LQG, we should treat some components of the metric tensor at the boundary as a quantum observable.*

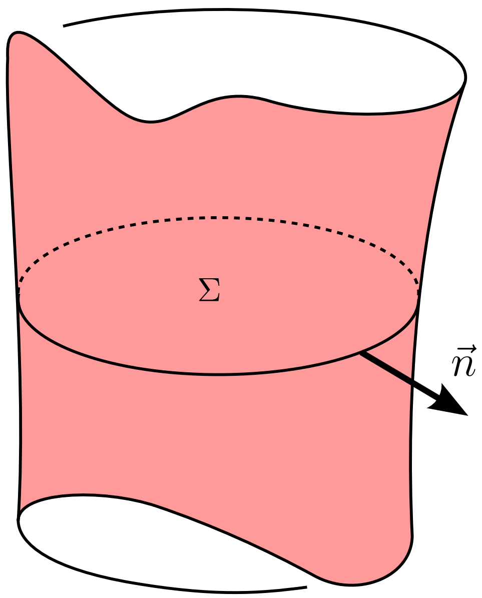
- *This excludes the usual Dirichlet boundary conditions (boundary metric fixed).*

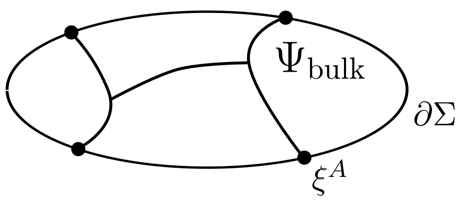
- *Conformal boundary conditions leave room to treat the conformal factor as a quantum observable. Conjugate variable (trace of the extrinsic curvature) fixed. Simplest choice: K=0 (the boundary is an extremal surface)*

$$\left.\begin{aligned} g_{\underleftarrow{ab}} = \Omega^2 q_{ab}, \quad \delta\Omega \neq 0, \\ \delta K = 0, \\ \delta q_{ab} = 0. \end{aligned}\right\} \text{conformal boundary conditions}$$

Spinors as boundary charges

- LQG Wilson lines excite a boundary charge, namely a spinor ξ^A.
  - Direction of the spinor determines the normal direction to the boundary.
  - Conformal factor turns into a composite field (norm of the spinor).





$$\vec{n} = \langle\xi|\vec{\sigma}|\xi\rangle / \langle\xi|\xi\rangle,$$

$$g_{\underleftarrow{ab}} = \underbrace{(4\pi G)^2 \langle\xi|\xi\rangle^2}_{\Omega^2} (m_a \bar{m}_b + \bar{m}_a m_b).$$

- The conformal boundary conditions turn into holomorphicity conditions for the loop gravity boundary spinors.

$$K = 0 \;\Leftrightarrow\; m^a \mathscr{D}_a \xi^A = 0,$$
$$\delta q_{ab} = 0 \;\Leftrightarrow\; \delta m_a = 0.$$

LQG boundary CFT

Unification of field equations and boundary conditions: Einstein equations in the bulk and boundary conditions derived from coupled bulk plus boundary action.

$$S[A,e|\xi] = \frac{1}{8\pi G}\int_{\mathscr{M}} e_i \wedge F^i[A] - \frac{\mathrm{i}}{\sqrt{2}}\int_{\mathscr{B}} \Big[\xi_A m \wedge D\xi^A - \mathrm{cc.}\Big]$$

- *No local degrees of freedom in the interior. Action defines boundary CFT with vanishing central charge.*
- *Infinite tower of quasi-local charges: Virasoro generators*
- *How does the boundary theory speak to LQG in the bulk?*

[see also talks by Dittrich and Seth on tuesday and Bonzom's talk on friday]

Boundary observables

Introduction of a boundary breaks diffeomorphism invariance.
Classification of diffeomorphisms:

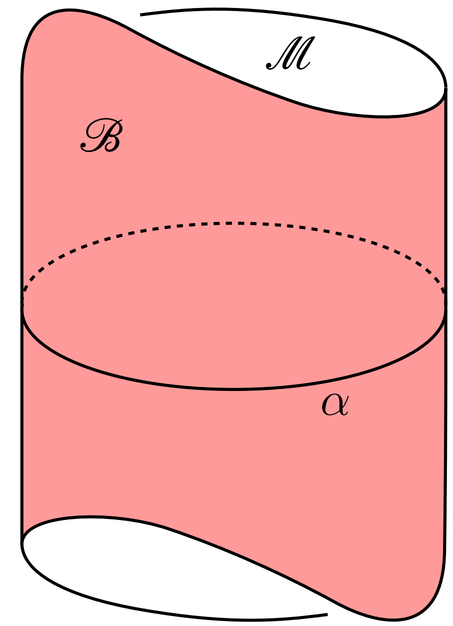
- *Small diffeomorphisms that vanish at the boundary are unphysical gauge transformations.*
- *Large diffeomorphism that move the boundary generate physical motions on phase space.*
- *Large diffeomorphisms that preserve the conformal structure of the boundary are true symmetries. The corresponding conserved Noether charges are the Virasoro generators.*

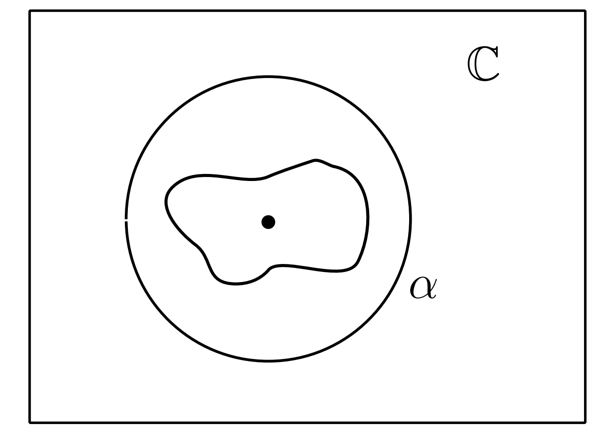
$$Q_t = -\frac{\mathrm{i}}{\sqrt{2}} \oint_{\mathscr{C}} \left[t^a m_a \xi_A \mathscr{D} \xi^A - \mathrm{cc.} \right] = \oint_{\mathscr{C}} dv^a t^b T_{ab}.$$

Which vacuum?

Using a mode expansion, we find two Virasoro algebras with c=0.

$$\left.\begin{aligned}\{L_m, L_n\} &= (m-n)L_{m+n},\\ \{\bar{L}_m, \bar{L}_n\} &= (m-n)\bar{L}_{m+n}.\end{aligned}\right\}\ \bar{L}_n = L_n^*$$





The quasi-local energy $H = L_0 + \bar{L}_0$ is unbounded from below. No surprise from GR perspective, since Brown—York quasi-local energy is not positive definite. There is no ground state of quasi-local energy. Choose different operator to select a vacuum state.

The length of a cross section defines a positive-definite quadratic form.

$$\mathrm{Length}[\alpha] = \oint_\alpha ds\,\Omega = 4\pi G \oint_\alpha ds\,\delta_{AA'}\xi^A\bar{\xi}^{A'} \geq 0.$$

CFT analogue of the AL vacuum

Using the mode expansion, we diagonalise the length operator for a given loop in terms of harmonic oscillators

$$\mathrm{Length}[\alpha] = 4\pi G \sum_{n=-\infty}^{\infty} \delta_{AA'} \bar{a}_n^{A'}[\alpha] a_n^A[\alpha]$$

Satisfy the Poisson commutation relations $\{a_n^A[\alpha], \bar{a}_m^{A'}[\alpha]\} = \mathrm{i}\delta_{mn}\delta^{AA'}$.

For any given loop , we define a corresponding vacuum $|0,\alpha\rangle : a_n^A[\alpha]|0,\alpha\rangle = 0$.

- *This is the CFT analogue of the Ashtekar—Lewandowski vacuum.*
- *A state of no geometry.*
- *Excitations over this vacuum represents quantised minimal surface boundaries (soap films) of different shape.*
- *Discrete spectrum of length recovered on the Hilbert space of the boundary CFT.*

… let us throttle down a bit…

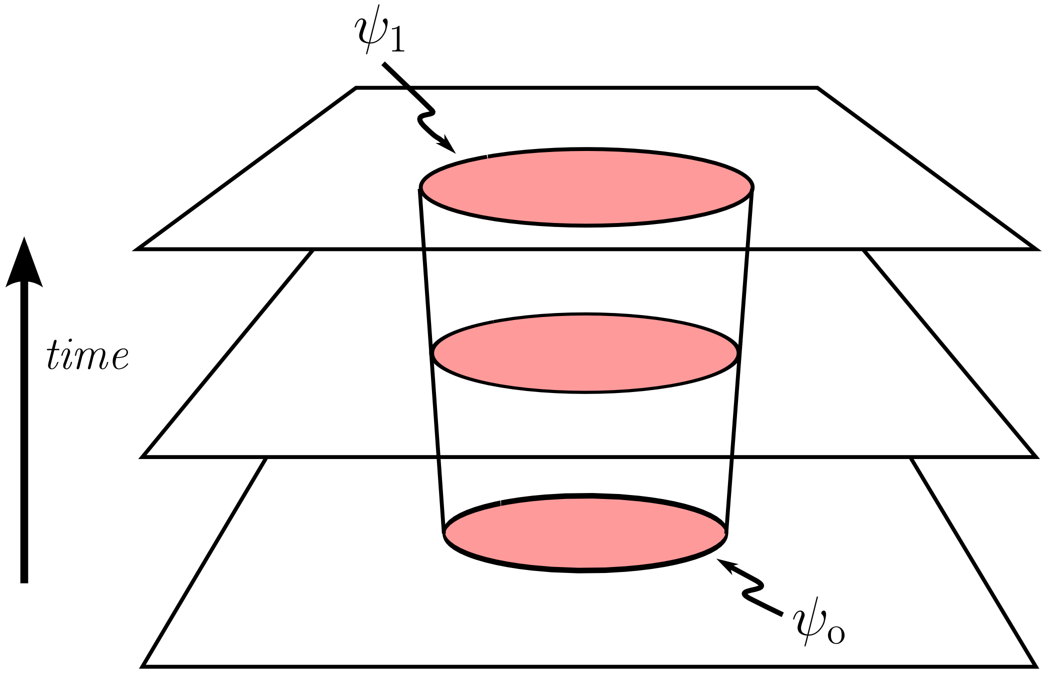
… what is this good for?

… what do we learn from this?

Four dimensions

Boundary amplitudes for edge states

- Lesson from 3d gravity
  - 3d gravity has no bulk degrees of freedom,
  - for given boundary conditions, we can integrate out the bulk, and are left with a field theory at the boundary.
  - Evolution is now encoded completely into transition amplitudes between edge states (quasi-local realisation of holography).

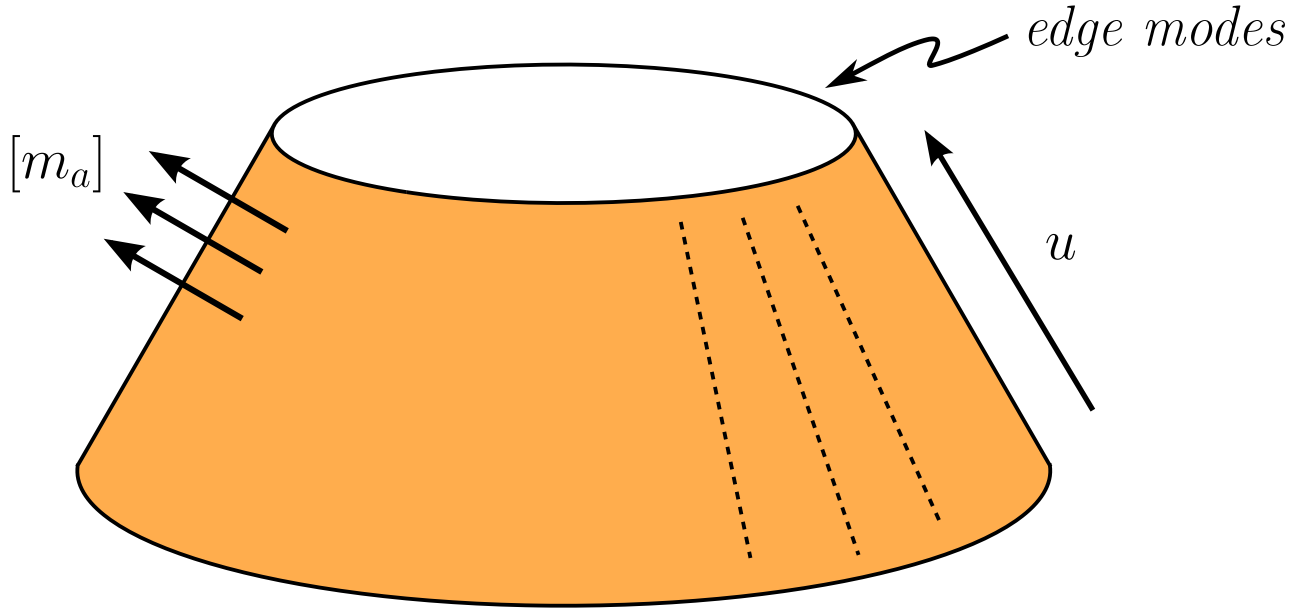


- Does a similar description exist in four Lorentzian dimensions?
  - *"Certainly not!", because gravity in four dimensions is very different from 3d gravity: there are now two degrees of freedom per point in the bulk!*
  - *"Possibly, yes", if we fix additional data along the boundary. Let us explore this possibility.*

Evolution equations for corner data

In general relativity, we can view the pull back of the Einstein equations to a null boundary (the constraint equations on a null hypersurface) as evolution equations for corner data (gravitational edge modes).

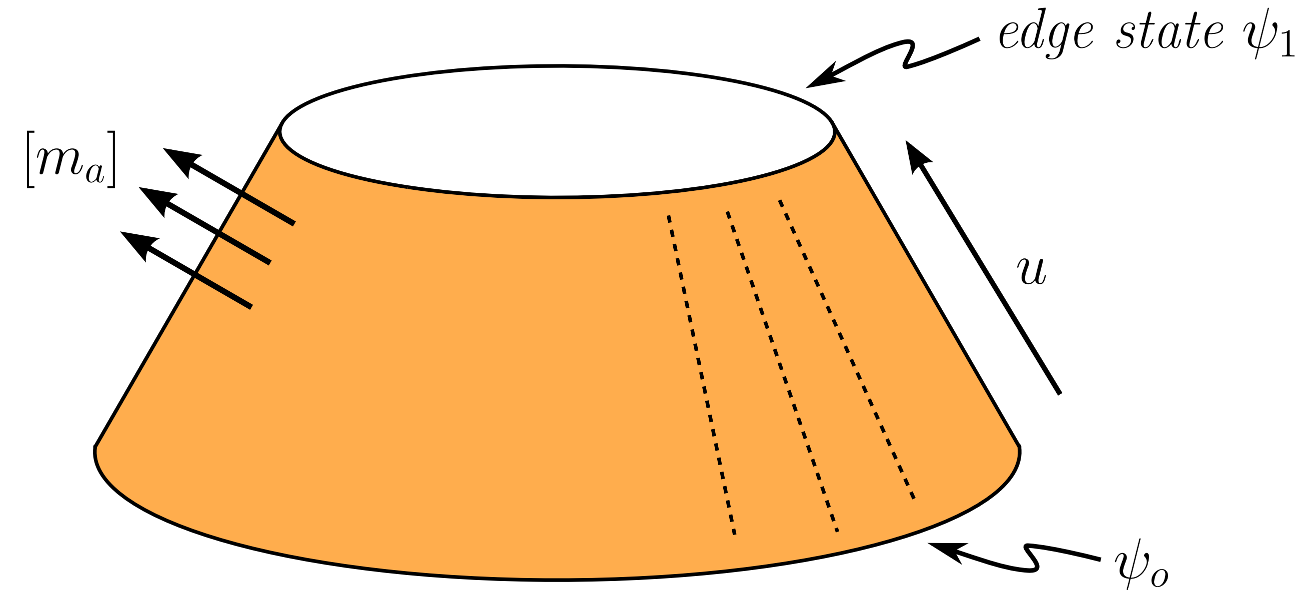
- free data along the null hypersurface: conformal equivalence class of 2d metrics $q_{ab} = 2m_{(a}\bar{m}_{b)}$.
- gauge conditions: non-affinity $\boldsymbol{\kappa}$, and choice of foliation of the null hypersurface (i.e. a choice of time variable *u*).
- free corner data (edge modes): conformal factor, out and ingoing expansion, outgoing shear, plus one additional spin coefficient (NP scalar $\boldsymbol{\tau}$).



[Bondi, Sachs, Winicour, Goldberg, Robinson, Soteriou, Reisenberger, …]

Boundary amplitudes as bulk states

In LQG we have a representation of gravitational edge states on a null boundary [ww].* What are the amplitudes for the edge states?



<u>Conjecture:</u> *From the perspective of QG in the bulk, the transition amplitudes for the edge states are state vectors (kets) on the radiative portion of the boundary Hilbert space.*

$$\Psi_{\text{bulk}}\big[\psi_o, \bar{\psi}_1, u, [m_a]\big] = \big\langle \psi_1 \big| U[u, m_a] \big| \psi_o \big\rangle,$$

$$\text{bulk state} = \text{transition amplitude for edge states.}$$

*see e.g. [ww] talk at loops'17, Warsaw.

Curvature and connection

- Intrinsic geometry of a null hypersurface can be fully characterised by´
  - the null flag of the boundary $\ell^A : \mathscr{N} \rightarrow \mathbb{C}^2$.
  - spinor-valued two-form $\eta_A = \big(\ell_A k - k_A m\big) \wedge \bar{m} \in \Omega^2(\mathscr{N} : \mathbb{C}^2)$.
- *Extrinsic geometry encoded into the pull-back of the self-dual connection to the null boundary (null analogue of the Ashtekar connection).*

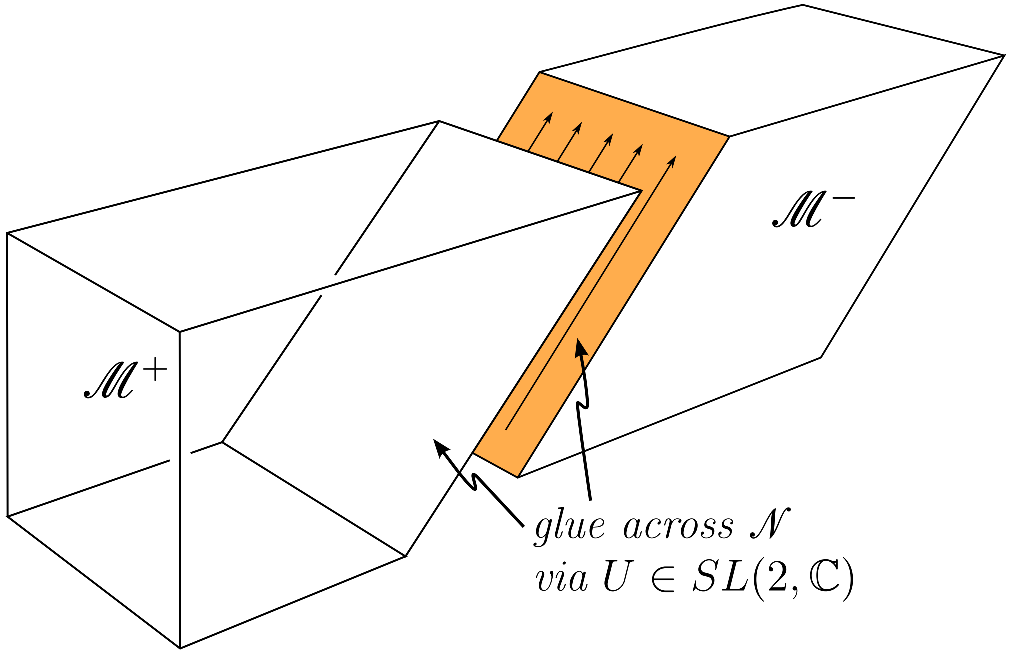
boundary is null	$\eta_{Aab}\ell_B = \frac{1}{2\mathrm{i}}\varepsilon_{ab}\epsilon_{AB} + \Sigma_{ABab}$
traceless part of Ricci tensor vanishes	$F^{AB}{}_{ab} = \frac{\Lambda}{3}\ell_{(A}\eta_{B)ab} + \underbrace{\psi_{ABC}}_{\text{Weyl part}}\eta^C{}_{ab}.$

NB: $\psi_{ABC} = {}^4\Psi_{ABCD}\ell^D,\ \Psi_0$ is invisible on $\mathscr{N}$.

Definition of the boundary action

Boundary action found by gluing two slabs of spacetime across a null boundary.

- Weyl spinor turns into a Lagrange multiplier for the gluing conditions.
- Defects (shockwaves) encoded into Maurer—Cartan form $\omega = U^{-1}(\mathrm{d}U + A^{+}U - UA^{-})$.



$$S\big[\eta_{Aab}, \ell^A, [A^{\pm}]^A{}_{Ba}, U^A{}_B, \varphi \big| [m_a]\big] =$$
$$= \frac{3\mathrm{i}}{8\pi\Lambda G}\int_{\mathscr{N}}\Big[\frac{\Lambda}{3}\eta_{(A}\ell_{B)}\wedge\omega^{AB} - \frac{\mathrm{i}}{2}\varphi\wedge\eta_A\ell^A + \psi_{ABC}\,\eta^A\wedge\omega^{BC} +$$
$$-\frac{1}{2}\mathrm{Tr}\big(F^+\wedge U\omega U^{-1}\big) - \frac{1}{2}\mathrm{Tr}\big(F^-\wedge\omega\big) + \frac{1}{6}\mathrm{Tr}\big(\omega\wedge\omega\wedge\omega\big)\Big] + \mathrm{cc.}$$

Boundary action = kinetic term for gravitational edge modes + reality conditions + gluing conditions for extrinsic curvature + dressed $SL^+(2,\mathbb{C})\times SL^-(2,\mathbb{C})$ Chern—Simons action.

Conclusion and outlook

… two aspects:

boundary kinematics & dynamics …

Quantum geometry in the continuum

Loop Gravity discreteness of space compatible with conventional Fock space quantization in the continuum.

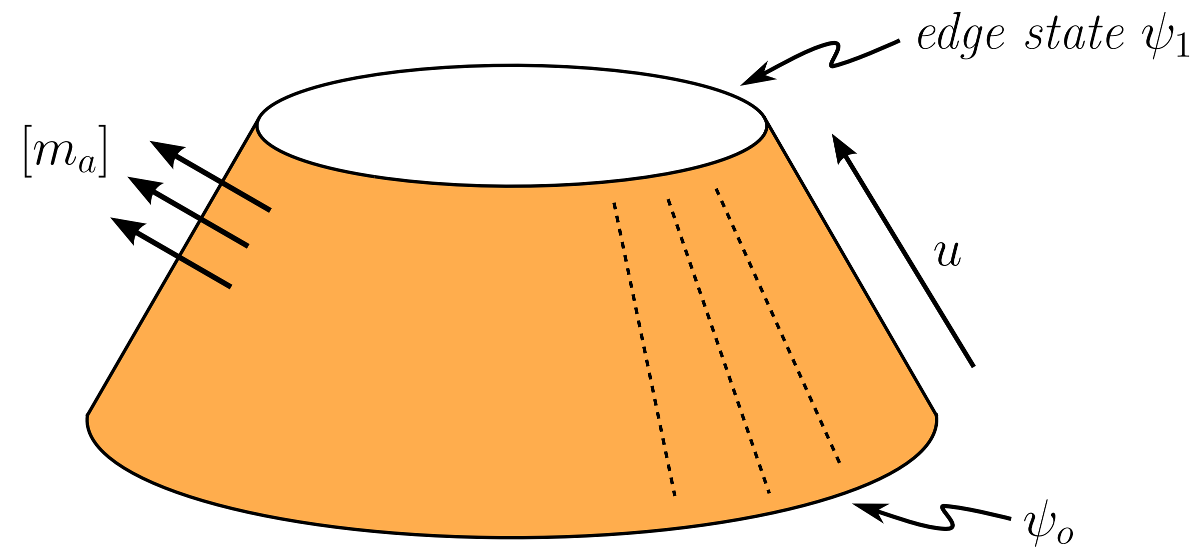
- One of the most celebrated and robust results of LQG so far: space itself has an atomic structure [Ashtekar, Rovelli, Smolin; Lewandowski, Thiemann].
- Derivation relied on an intermediate step, namely an auxiliary lattice.
- This raised concern and criticism from the wider high-energy/strings community.
  - Discreteness built in from the onset by choice of lattice variables?
  - Minimal length at odds with Lorentz invariance?
- The quantum discreteness of space can be understood without this intermediate step [1,2,3] directly from the quantisation of gravitational edge modes in the continuum in both 2+1 (Euclidean) and 3+1 (Lorentzian) dimensions.

[1] w.wieland, New boundary variables for classical and quantum gravity […], **Class. Quantum Grav. 34 (2017)**]

[2] w.wieland, Fock representation of gravitational boundary modes […], **Annales Henri Poincaré 18 (2017)**]

[3] w.wieland, Conformal boundary conditions, loop gravity and the continuum, **JHEP (2018):89**]

Dynamics of gravitational edge modes



We now have a proposal for the boundary field theory for the gravitational edge modes on a null surface.

Key future applications:

1. **Scattering of gravitational edge modes along null surfaces.**
2. **Generalisation to Einstein—Yang—Mills.**
3. **Boundary observables and black hole spectroscopy:** Establish quantisation of energy, angular momentum, shear and expansion via quantisation of the boundary field theory.
4. **Symmetry reduction:** In k=0 LQC, we quantise symmetry reduced GR in a fiducial co-moving box. If we replace the fiducial box by the interior of a lightcone, we should be able to understand LQC from the quantisation of a symmetry reduced version of the boundary field theory alone → scale factor identified with the square root of the cross-sectional area. Potential connection to Abhay's talk on Tuesday.