Quasi-local charges and LQG

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… in gravity …

… what is the system to be quantised?

The boundary is part of the system

- The simplest Dirac observables are the charge integrals at infinity
  - Linear Momentum: $E_\xi = \frac{1}{8\pi G} \lim_{\rho\to\infty} \oint_{S^2_\rho} d^2\Omega\, \rho^3 E_{ab} n^b \xi^a$
  - Angular Momentum: $J_\omega = \frac{1}{16\pi G} \lim_{\rho\to\infty} \oint_{S^2_\rho} d^2\Omega\, \rho^4 B^{[a}{}_c \hat{\rho}^{b]} n^c \omega_{ab}$
- The first law links charges at infinity to the BH area at finite distance

$$\underbrace{\delta M + \Omega\,\delta J}_{\text{at infinity}} = \frac{\kappa}{8\pi G}\delta A$$

➡ The black hole exterior is a Hamiltonian system in a box!

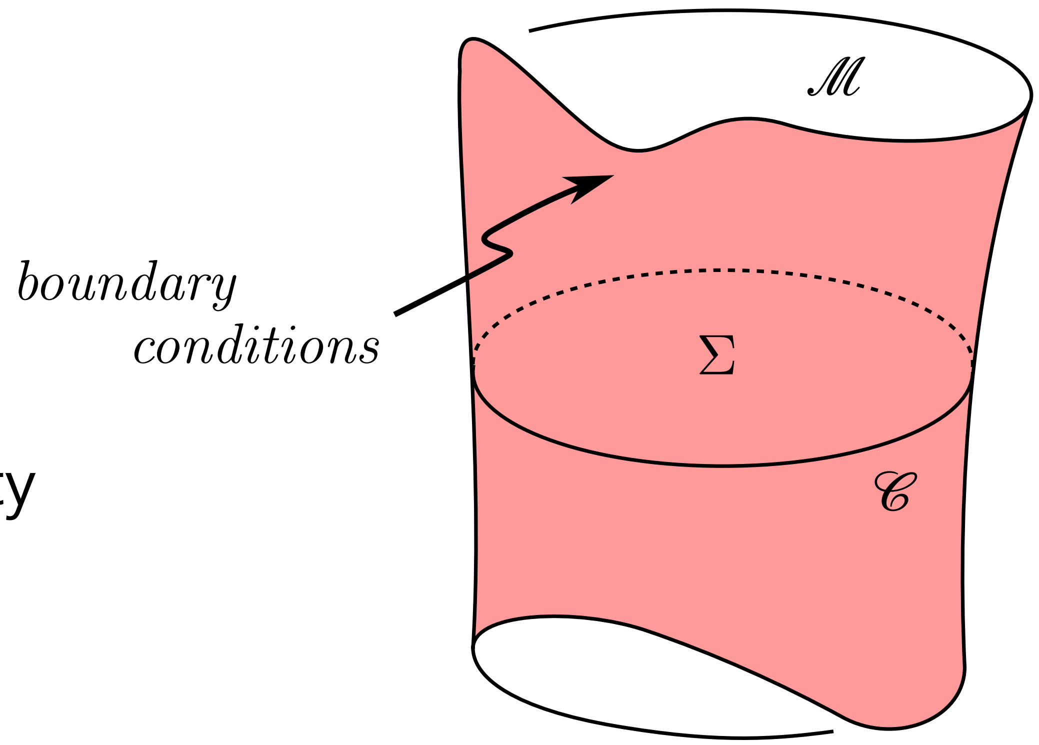
  - Boundary at infinity: *asymptotic charges*
  - Black hole horizon: *area, surface gravity, edge modes*

… let us first

understand the problem in 3d LQG…

Quasi-local quantisation of 3d gravity

- Setup: three-dimensional euclidean gravity, vanishing cosmological constant.



- Quasi-local approach: Quantise gravity in a box (finite cylinder).

- Which box? Shape of the box is itself dynamical: depends on the boundary conditions/boundary dynamics.

Boundary CFT in spin network representation: [Dittrich, Geiller, Goeller, Riello, Bonzom, Livine, Perez, Pranzetti, Freidel]

Conformal boundary conditions

Different boxes ~ different boundary conditions ~ different dual boundary field theories.

- *To compare the boundary field theory with LQG, we should treat some components of the metric tensor at the boundary as a quantum observable.*

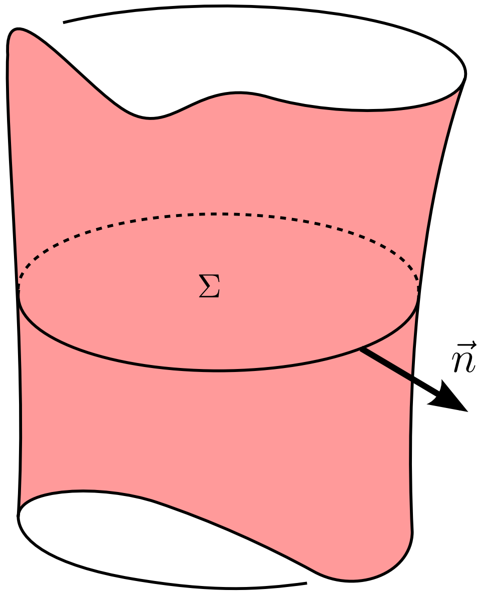
- *This excludes the usual Dirichlet boundary conditions (boundary metric fixed).*

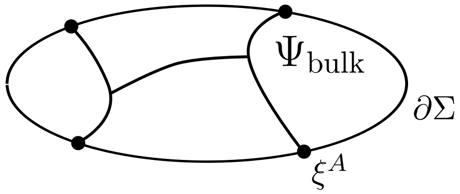
- *Conformal boundary conditions leave room to treat the conformal factor as a quantum observable. Conjugate variable (trace of the extrinsic curvature) fixed. Simplest choice: K=0 (the boundary is an extremal surface)*

$$\left.\begin{aligned} g_{\underleftarrow{ab}} = \Omega^2 q_{ab}, \quad \delta\Omega \neq 0, \\ \delta K = 0, \\ \delta q_{ab} = 0. \end{aligned}\right\}$$ *conformal boundary conditions*

Spinors as boundary charges

- LQG Wilson lines excite a boundary charge, namely a spinor ξ^A.
  - Direction of the spinor determines the normal direction to the boundary.
  - Conformal factor turns into a composite field (norm of the spinor).





$$\vec{n} = \langle\xi|\vec{\sigma}|\xi\rangle / \langle\xi|\xi\rangle,$$

$$g_{\underleftarrow{ab}} = \underbrace{(4\pi G)^2\langle\xi|\xi\rangle^2}_{\Omega^2}(m_a\bar{m}_b + \bar{m}_a m_b).$$

- The conformal boundary conditions turn into holomorphicity conditions for the loop gravity boundary spinors.

$$K = 0 \;\Leftrightarrow\; m^a\mathscr{D}_a\xi^A = 0,$$
$$\delta q_{ab} = 0 \;\Leftrightarrow\; \delta m_a = 0.$$

LQG boundary CFT

Unification of field equations and boundary conditions: Einstein equations in the bulk and boundary conditions derived from coupled bulk plus boundary action.

$$S[A, e|\xi] = \frac{1}{8\pi G}\int_{\mathscr{M}} e_i \wedge F^i[A] - \frac{\mathrm{i}}{\sqrt{2}}\int_{\mathscr{B}} \Big[\xi_A m \wedge D\xi^A - \mathrm{cc.}\Big]$$

- *No local degrees of freedom in the interior. Action defines boundary CFT with vanishing central charge.*
- *Infinite tower of quasi-local charges: Virasoro generators*
- *How does the boundary theory speak to LQG in the bulk?*

[see also talks by Dittrich and Seth on tuesday and Bonzom's talk on friday]

Boundary observables

Introduction of a boundary breaks diffeomorphism invariance.
Classification of diffeomorphisms:

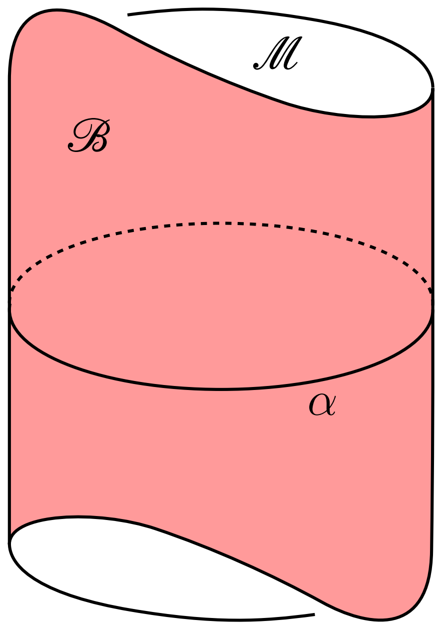
- *Small diffeomorphisms that vanish at the boundary are unphysical gauge transformations.*
- *Large diffeomorphism that move the boundary generate physical motions on phase space.*
- *Large diffeomorphisms that preserve the conformal structure of the boundary are true symmetries. The corresponding conserved Noether charges are the Virasoro generators.*

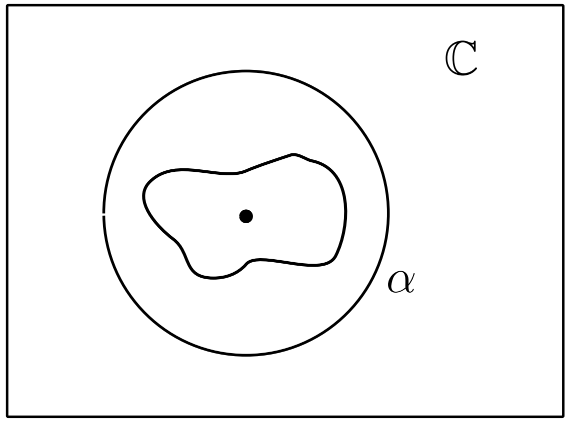
$$Q_t = -\frac{\mathrm{i}}{\sqrt{2}} \oint_{\mathscr{C}} \Big[t^a m_a \xi_A \mathscr{D} \xi^A - \mathrm{cc.} \Big] = \oint_{\mathscr{C}} dv^a t^b T_{ab}.$$

Which vacuum?

Using a mode expansion, we find two Virasoro algebras with c=0.

$$\left.\begin{aligned}\{L_m, L_n\} &= (m-n)L_{m+n},\\ \{\bar{L}_m, \bar{L}_n\} &= (m-n)\bar{L}_{m+n}.\end{aligned}\right\}\ \bar{L}_n = L_n^*$$





The quasi-local energy $H = L_0 + \bar{L}_0$ is unbounded from below. No surprise from GR perspective, since Brown—York quasi-local energy is not positive definite. Choose different operator to select a vacuum state.

The length of a cross section defines a positive-definite quadratic form.

$$\mathrm{Length}[\alpha] = \oint_\alpha ds\,\Omega = 4\pi G \oint_\alpha ds\,\delta_{AA'}\xi^A\bar{\xi}^{A'} \geq 0.$$

CFT analogue of the AL vacuum

Using the mode expansion, we diagonalise the length operator for a given loop in terms of harmonic oscillators

$$\mathrm{Length}[\alpha] = 4\pi G \sum_{n=-\infty}^{\infty} \delta_{AA'} \bar{a}_n^{A'}[\alpha] a_n^A[\alpha]$$

Satisfy the Poisson commutation relations $\{a_n^A[\alpha], \bar{a}_m^{A'}[\alpha]\} = \mathrm{i}\delta_{mn}\delta^{AA'}$.

For any given loop , we define a corresponding vacuum $|0, \alpha\rangle : \, a_n^A[\alpha]|0, \alpha\rangle = 0.$

- *This is the CFT analogue of the Ashtekar—Lewandowski vacuum.*
- *A state of no geometry.*
- *Excitations over this vacuum represent quantised minimal surface boundaries (soap films) of different shape.*
- *Discrete spectrum of length recovered on the Hilbert space of the boundary CFT.*

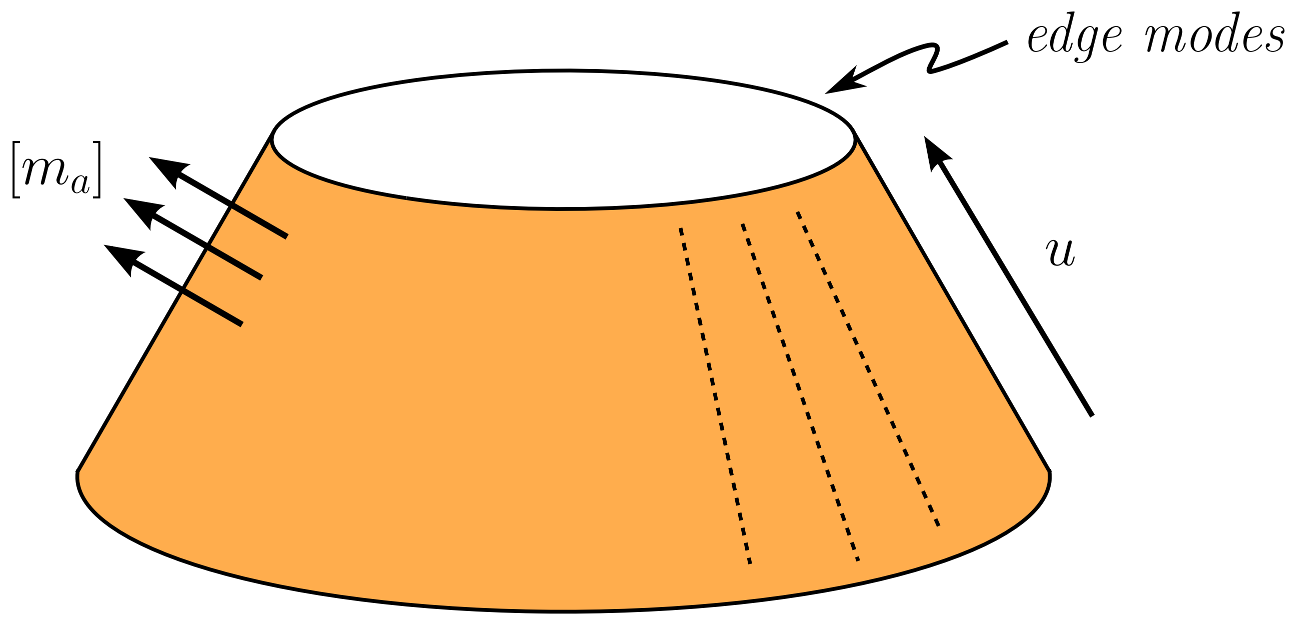
… is there a generalisation to four dimensions?

- *"Certainly not!", because gravity in four dimensions is very different from 3d gravity: there are now two degrees of freedom per point in the bulk!*
- *"Possibly, yes", if we fix additional data along the boundary. Let us explore this possibility.*

Evolution equations for corner data

In general relativity, we can view the pull back of the Einstein equations to a null boundary (the constraint equations on a null hypersurface) as evolution equations for corner data (gravitational edge modes).

- free data along the null hypersurface: conformal equivalence class of 2d metrics $q_{ab} = 2m_{(a}\bar{m}_{b)}$.
- gauge conditions: non-affinity $\boldsymbol{\kappa}$, and choice of foliation of the null hypersurface (i.e. a choice of time variable u).
- free corner data (edge modes): conformal factor, out and ingoing expansion, outgoing shear, plus one additional spin coefficient (NP scalar $\boldsymbol{\tau}$).

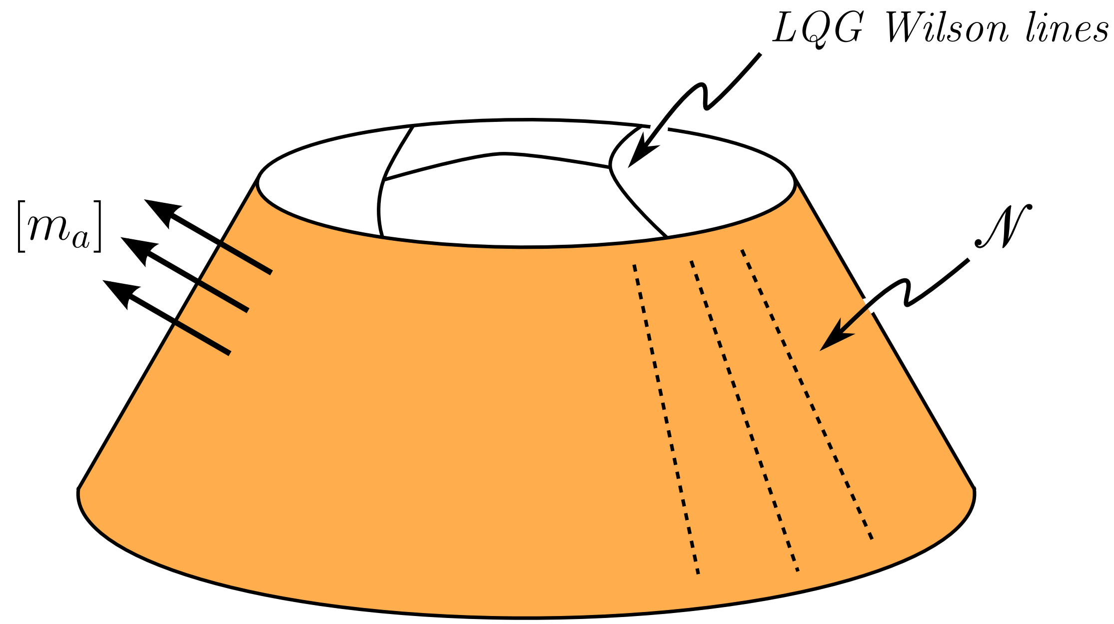


[Bondi, Sachs, Winicour, Goldberg, Robinson, Soteriou, Reisenberger, …]

Bulk plus boundary action

- Intrinsic null geometry encoded into boundary fields
  - null flag of the boundary $\ell^A \in \mathbb{C}^2$.
  - two-form $\eta_A = \big(\ell_A k - k_A m\big) \wedge \bar{m}$.
- Conformal boundary conditions

$$k = -\mathrm{d}u, \quad \delta k = 0, \quad \delta m \propto m.$$



$$S\big[A, e, \eta, \ell \big| [m_a]\big] = \frac{\mathrm{i}}{8\pi G}\Big[\int_{\mathscr{M}} \Sigma_{AB} \wedge F^{AB} + \int_{\mathscr{N}} \eta_A \wedge D\ell^A\Big] + \mathrm{cc}.$$

Symplectic structure, charges, area

- Symplectic structure has bulk plus boundary terms

$$\Theta_\Sigma = \frac{\mathrm{i}}{8\pi G}\left[\int_\Sigma \Sigma_{AB}\wedge \mathbb{d}A^{AB} + \oint_{\partial\Sigma}\eta_A \mathbb{d}\ell^A\right] + \mathrm{cc.}$$

- Charges obtained by Integrating Hamilton's equations $\Omega_\Sigma(X_Q,\delta) = -\delta[Q].$

- Area-flux quantisation follows from quantisation of the U(1) boundary charge.

complexified U(1) generators	$Q_f[\mathscr{C}] = \frac{\mathrm{i}}{8\pi G}\oint_{\mathscr{C}}(f\,\eta_A\ell^A - \mathrm{cc.}),$
tangential diffeos	$J_\xi[\mathscr{C}] = \frac{\mathrm{i}}{8\pi G}\oint_{\mathscr{C}}(\eta_A\,\xi^a D_a\ell^A - \mathrm{cc.}).$

… summary:

boundary kinematics & dynamics …

Quantum geometry in the continuum

Loop Gravity discreteness of space compatible with conventional Fock space quantization in the continuum.

- One of the most celebrated and robust results of LQG so far: space itself has an atomic structure [Ashtekar, Rovelli, Smolin; Lewandowski, Thiemann].
- Derivation relied on an intermediate step, namely an auxiliary lattice.
- This raised concern and criticism from the wider high-energy/strings community.
  - Discreteness built in from the onset by choice of lattice variables?
  - Minimal length at odds with Lorentz invariance?
- The quantum discreteness of space can be understood without this intermediate step [1,2,3] directly from the quantisation of gravitational edge modes in the continuum in both 2+1 (Euclidean) and 3+1 (Lorentzian) dimensions.
- At the boundary, an infinite tower of quasi-local boundary observables is found.

[1] **w.wieland**, New boundary variables for classical and quantum gravity […], **Class. Quantum Grav. 34 (2017)**]

[2] **w.wieland**, Fock representation of gravitational boundary modes […], **Annales Henri Poincaré 18 (2017)**]

[3] **w.wieland**, Conformal boundary conditions, loop gravity and the continuum, **JHEP (2018):89**]