Subsystems in classical and quantum gravity, 16. Aug. 2023

Outline

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2. Lesson from perturbative gravity
3. Which regions? — the case for null boundaries
4. Discussion

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2302.11629
2104.05803
2012.01889

1. Motivation

- Holography gives us one state

$$Z^{T\bar{T}}[g_{ab}, \phi] = \Psi_{WdW}[g_{ab}, \phi]$$

it seems infinitely many Dirac observables are set to zero here:

$$X^S \Psi_{WdW} = 0 \quad ;$$

$$[\Pi_\sigma, X^S] \neq 0 \quad :$$

new states

$$\Pi_\sigma \Psi_{WdW} = |\sigma\rangle$$

we want all states

not just that

• in quantum gravity, we expect all metric components to be quantum

$$ds^2 = -2\,du\,dv + r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2) + \\ + \ldots$$

$$u = \frac{1}{\sqrt{2}}(t-r)$$
$$v = \frac{1}{\sqrt{2}}(t+r)$$

$v = \frac{1}{\sqrt{2}}(t+r) \longrightarrow \infty$ asymptotic bound

in conflict (?) with Heisenberg relations

$\hat{v}$ is itself operator / edge mode / quantum reference frame ...

$\hat{v}|\psi\rangle \longrightarrow \infty$

this is what effectively happens in 2+1, where we only have boundary modes

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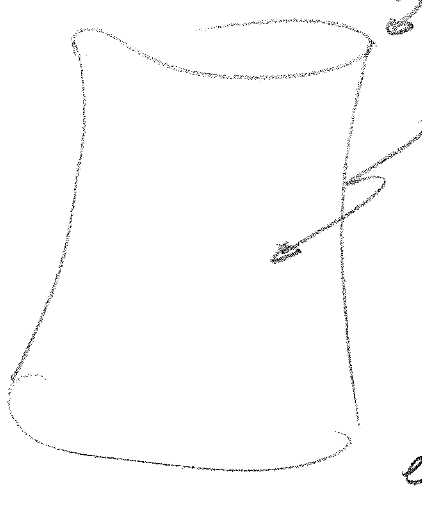
- basic idea: finite regions are the elementary building blocks – atoms of geometry.
- whether region is large or small itself determined by the quantum state.
- setup: definition of the dynamics involves prescription (boundary conditions) for how to evolve the boundary into world tube



data here – edge modes

data here
- radiative modes
- choice of surface, e.g. $t = \text{const}$

example: $u^\mu = \dfrac{x^\mu}{\sqrt{-x_\mu x^\mu}} = \dfrac{x^\mu}{\rho}$

$$\partial_\mu u^\mu = \frac{4-1}{\sqrt{-x_\mu x^\mu}} = \frac{3}{\rho}$$

2. Lesson from perturbative gravity

- dealing with boundaries & finite regions, first-order formalism is more advantageous than metric gravity
    - Stokes's theorem rather than Gauß, no need to distinguish signature of boundary
    - more primitive notion of projection, pull-back
    - can deal with situations when metric is degenerate
    - polynomial constraints

- action

$$S_{EHP}[e,\omega] = \frac{1}{16\pi G}\int_M *(e_\alpha \wedge e_\beta) \wedge F^{\alpha\beta}[A]$$

- α, β ... internal indices
- $*$ ... Hodge dual in internal space
- $F = dA + \frac{1}{2}[A,A]$ ... curvature

- symplectic structure

$$e^\alpha = \Lambda^\alpha{}_\mu \left(dX^\mu + f^\mu \right)$$

satisfy gauge-fixing conditions

$$A^\alpha{}_\beta = \Lambda^\alpha{}_\mu \, d\Lambda_\beta{}^\mu + \Lambda^\alpha{}_\mu \Delta^\mu{}_\nu \Lambda_\beta{}^\nu$$

$$f^\mu = \sum_n (8\pi G)^{\frac{n}{2}} \, {}^{(n)}f^\mu$$

$$\Delta^\mu{}_\nu = \sum_n (8\pi G)^{\frac{n}{2}} \, {}^{(n)}\Delta^\mu{}_\nu$$

- Maurer - Cartan forms, ghosts

$$\mathbb{X}^a = \left[\frac{\partial}{\partial x^\mu}\right]^a \mathrm{d}\mathrm{X}^\mu \; \ldots \; \text{diffeo-ghost}$$

$$\mathbb{M}^\mu{}_\nu = \mathrm{d}\Lambda_o{}^\kappa \Lambda^\nu{}_\kappa \; \ldots \; \text{Lorentz-ghost}$$

- covariant field space derivative

$$\mathbb{D} f^\mu = \mathrm{d} f^\mu - \mathcal{L}_{\mathbb{X}} f^\mu$$

- bulk and boundary symplectic structure for $\sqrt{8\pi G} \to 0$ in finite dist

$$\Omega_{\mathcal{D}} = \Omega_{\mathcal{D}}^{\text{red}} + \Omega_{\partial\mathcal{D}} + \mathcal{O}(\sqrt{8\pi G})$$

$$\Omega^{rad}_{\mathcal{D}} = \int_{\mathcal{D}} *\left(dX_{[\mu} \wedge \mathbb{D}^{(1)} f_{\nu]}\right) \wedge \mathbb{D}^{(1)} \Delta^{\mu\nu}$$

$$\Omega_{\partial\mathcal{D}} = \oint_{\partial\mathcal{D}} dP_\mu \, dX^\mu \; +$$

$$-\frac{1}{2}\oint_{\partial\mathcal{D}} d\left(S^\mu{}_\nu \, d\Lambda_\mu{}^\alpha \, \Lambda^\nu{}_\alpha\right)$$

$P_\mu = \varphi^*_{\partial\mathcal{D}} *^{(2)}\Delta_{\mu\nu} \wedge dX^\nu$ .... momentum current

$$S_{\mu\nu} = \frac{1}{8\pi G}\varphi^*_{\partial\mathcal{D}}\Big[*\left(dX_{[\mu} \wedge dX_{\nu]}\right) +$$

flat space contribution

$$+\,16\pi G *\left(dX_{[\mu} \wedge {}^{(2)}f_{\nu]}\right)\Big] \ldots$$

.... spin current

$J_{\mu\nu} = 2P_{[\mu}X_{\nu]} + S_{\mu\nu}$ .... angular momentum current

L

limit to asymptotic boundary

$$\rho = \sqrt{-x_\mu x^\mu} \longrightarrow \infty$$

but

$$\delta x^\mu \big|_{\partial\Omega} = Q^\mu(\vartheta, \varphi) \quad \text{finite}$$

dual to BMS charges

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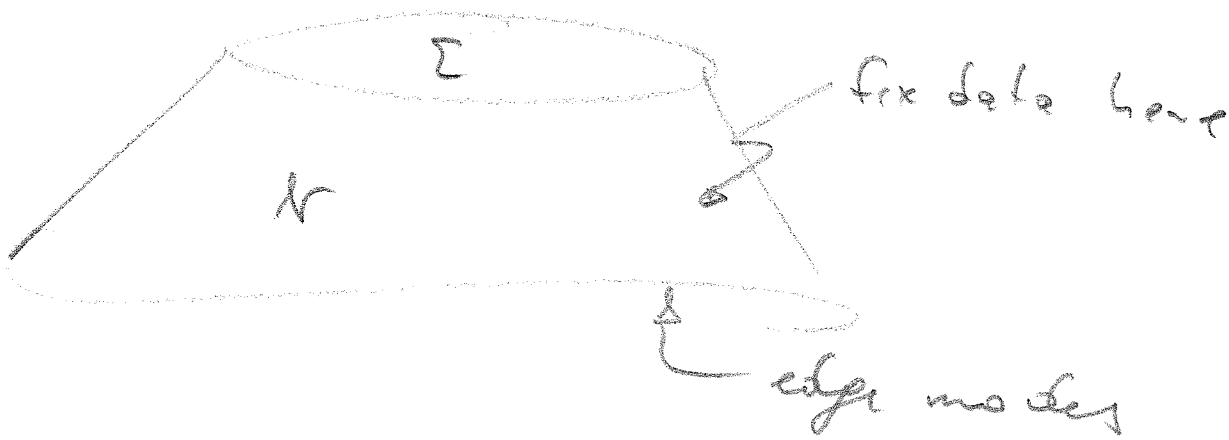
edge modes / boundary gravitons are just the different ways to choose boundary / in case of asymptotic boundary how to embed it into physical spacetime

3. Which regions? — the case for null surface boundaries



<u>self-dual variables</u>

$e_{AA'}$ .... tetrad

$A^A{}_B$ ... spin connection ; $A^A{}_A = 0$

$\ell^a = i\ell^A \bar{\ell}^{A'}$ .... null vector = square of spinor

$e_{AA'} \wedge e_{BB'} = -\bar{\epsilon}_{A'B'} \Sigma_{AB} - \epsilon_{AB} \bar{\Sigma}_{A'B'}$

$*\Sigma_{AB} = i \Sigma_{AB}$ .... self-dual two-form

bulk action

$$S_M[A^A{}_B, e_{AA'}] = \frac{i}{8\pi\gamma G}(\gamma + i)\int_M \Big(\Sigma_{AB} \wedge F^{AB} + - \frac{\Lambda}{6}\Sigma_{AB} \wedge \Sigma^{AB}\Big) + c.c.$$

boundary action

$$S_N[e_A, \ell^A, A^A{}_B \,|\, m, A, \vartheta] =$$

$$= \frac{i}{8\pi\gamma G}(\gamma + i)\int_N \Big(e_A \wedge \bar{m} \wedge (D - \tfrac{1}{2}A)\ell^A + - \frac{\vartheta}{4} e_A \wedge e^A \wedge \bar{m}\Big) + c.c.$$

boundary conditions

$$g = [u, A, \vartheta]/_{\sim}\ , \quad \delta g = 0 \ \text{ on } N$$

$$[u, A, \vartheta] \sim [e^{i\phi} u, A + i\,d\phi, \vartheta]$$

$$[u, A, \vartheta] \sim [u, A + d\lambda, e^{\lambda}\vartheta]$$

$$[u, A, \vartheta] \sim [e^{f} u, A, e^{f}\vartheta]$$

\+ diffeomorphisms of N

real

\+ shifts $A \longrightarrow A + \frac{i}{2}\frac{\alpha}{\zeta + i} \longrightarrow$

$$\longrightarrow A + \zeta\bar{u} + \bar{\zeta} u$$

physical phase space

how to parametrise this equivalence class?

$$m_a = \Omega\,(S \triangleright m_a^{(0)})$$

SL(2, R)

conformal factor

$$m_a^{(0)} = \frac{\partial_a z}{1+|z|^2}\;;\quad d^2 v_0 = -i\, m^{(0)} \wedge \bar{m}^{(0)}$$

choice of time

$$U^b \nabla_b U^a = -\frac{1}{2}\left(\Omega^{-2} \frac{d}{du}\Omega^2\right) U^a$$

$$U\big|_{\partial\Sigma} = \pm 1$$

$SL(2,\mathbb{R})$ Maurer-Cartan form

$$dU \frac{d}{dU} S \cdot S^{-1} = \tilde{\varphi} J + \underset{\substack{\uparrow \\ \text{"graviton"}}}{\tilde{\eta}} \bar{X} + \bar{\tilde{\eta}} X$$

symplectic structure

$$\Theta_N = \frac{1}{8\pi G} \int_N d^2 v_o \wedge \Big[p_K \, d\tilde{K} + \gamma^{-1} E \, d\tilde{\Phi} + \underbrace{\tilde{\Pi}^l{}_m \wedge [S \, dS^{-1}]^m{}_l}_{T^* SL(2,\mathbb{R})} \Big]$$

$E = \Omega^2$ ... area/conformal factor

$\tilde{K} = dU$ ... time

$p_K = \frac{d}{dU} E$ ... expansion

constraints

decomposition of $SL(2,\mathbb{R})$ in $U(1)$ and boosts

$$\tilde{\Pi}^l{}_m = \tilde{I}\, 1^l{}_m + \tilde{\pi}\, \bar{X}^l{}_m + \tilde{\bar{\pi}}\, X^l{}_m$$

$\tilde{I} - \frac{1}{2\gamma} dU \frac{d}{dU} E = 0$ ($U(1)$ part of $\tilde{\Pi}^l{}_m$ is the expansion)

$p_t \tilde{K} - dU \frac{d}{dU} E = 0$

$\tilde{\Phi} - \tilde{\varphi} = 0$ ($U(1)$ angle of $\dot{S}S^{-1}$ is momentum conjugate of E)

$\Omega^{-1} \tilde{\pi} - \frac{\gamma + i}{\gamma} \Omega h = 0$

$\frac{1}{2} \frac{d^2}{dU^2} \Omega^2 + \sigma\bar{\sigma} = 0$

$\tilde{\pi} = \frac{\gamma + i}{\gamma} \Omega \sigma\, dU$

Dirac bracket for radiative modes

$$\{ S^{i}{}_{m}(x), S^{\bar{j}}{}_{n}(y) \}^{*} =$$

$$= -4\pi G \, \Theta(x,y) \, \delta^{(2)}(x^{\flat}, y^{\flat}) \, \Omega^{-1}(x) \, \Omega^{-1}(y) \Big[e^{-2i(\Delta(x) - \Delta(y))} [X S(x)]^{i}{}_{m} \cdot [\bar{X} S(y)]^{\bar{j}}{}_{n} + c.c. \Big]$$

with $U(1)$ holonomy

$$\Delta(x) = \int_{\gamma_x} \tilde{\Phi} \quad \ldots \text{ along null generator until } x$$