Loop Quantum Gravity and Quantization of Null Surfaces

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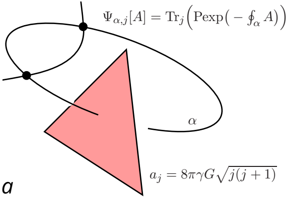
Introduction

Quantum geometries vs. discrete geometries

- LQG is a quantisation of the spin structure of spacetime:
  - *Quantum states of gravity are built from gravitational Wilson lines for the spin connection.*
  - *Each Wilson line carries a spin j, which represents a quantum of area.*

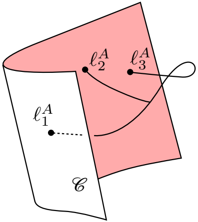


- The appearance of such discrete and combinatorial structures has led to the idea that LQG is a fundamentally discrete theory on a spacetime lattice. The *'full theory'* would only be defined through a continuum limit. It was then often argued that the discrete spectra are lattice artefacts that may disappear in the continuum limit.

- In this talk, I will argue that the LQG discreteness of geometry already appears in the continuum and can be understood from the quantisation of gravitational edge modes on a null surface. Spin networks or triangulations of space do not enter the construction.

Boundaries in LQG

- **Basic question:** Bulk geometry described by spin networks. If they hit a boundary, a surface charge is excited (namely a spinor). What is the classical Hamiltonian description for these loop gravity boundary spinors? What is their role in classical GR?



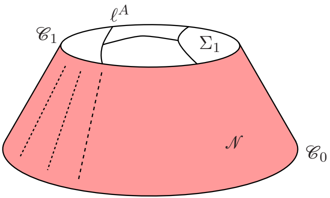
- **Spinors as square root of geometry:** A spinor $\psi \in \mathbb{C}^2$ is the square root of a null direction $(T, X, Y, Z) : T^2 - X^2 - Y^2 - Z^2 = 0$

$$\psi \otimes \bar{\psi} = \frac{1}{\sqrt{2}} \begin{pmatrix} T+Z & X-\mathrm{i}Z \\ X+\mathrm{i}Y & T-Z \end{pmatrix}, \qquad \psi \to \mathrm{e}^{\frac{\mathrm{i}}{2}\varphi}\psi.$$

- **Suggestive idea:** The loop quantum gravity boundary spinors encode gravitational edge modes on a null boundary.
  - *Part of a wider effort to realise soft (edge) modes, memory effect, quasi-local observables within non-perturbative QG, and establish connections to other QG approaches (string theory, quasi-local holography [Jacobson], Strominger's soft modes, tensor networks) [L. Freidel, M. Geiller, A. Riello, D. Pranzetti, B. Dittrich, W. Donnelly]*
  - *Emerged out of LQG bosonic representation [L. Freidel, S. Speziale, E. Livine, ww, E. Bianchi et al.]*

Spinors as gravitational edge modes on a null surface

Subsystems of the gravitational field with inner null boundaries $\mathscr{N}$ (all fields assumed to be regular on $\mathscr{N}$, excluding e.g. focal points).



- Boundary consists of partial Cauchy surfaces Σ_0, Σ_1
- and a null surface $\mathscr{N}$ (e.g. isolated Horizon, but this is not necessary).

- The gravitational action consists of bulk plus boundary contributions.
- What counter term shall we put at $\mathscr{N}$? Difficulty: there is now an additional constraint to be imposed—that the boundary is null.
- Working with self-dual Ashtekar variables in the bulk, we will find such a boundary term in terms of boundary spinors coupled to the spin connection in the bulk.

*R. Wald and A. Zoupas, A General Definition of "Conserved Quantities" in General Relativity and Other Theories of Gravity, Phys.Rev. D 61 (2000), arXiv::gr-qc/9911095.
*T. Andrade and D. Marolf, Asymptotic symmetries from finite boxes, Class. Quant. Gravity. 33 (2016), arXiv:1508.02515.

On a null surface it is useful to work with forms rather than vectors. Given a tetrad e^α, we have a hierarchy of p-forms: $e^{\alpha_1} \wedge \cdots \wedge e^{\alpha_p}$.

- Plebański's directed area two-form $\Sigma^{\alpha\beta} = e^\alpha \wedge e^\beta$ splits into self-dual and anti-selfdual components:

$$\begin{pmatrix} \Sigma^A{}_B & \emptyset \\ \emptyset & -\bar{\Sigma}_{A'}{}^{B'} \end{pmatrix} = -\frac{1}{8}[\gamma_\alpha, \gamma_\beta] e^\alpha \wedge e^\beta .$$

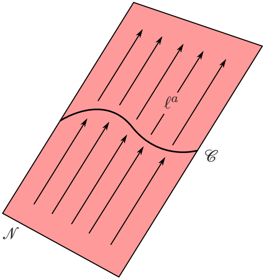
- *On a null surface $\mathcal{N}$, there always exists a spinor $\ell^A : \mathcal{N} \to \mathbb{C}^2$ and a spinor-valued two-form $\eta^A{}_{ab} \in \Omega^2(\mathcal{N} : \mathbb{C}^2)$ such that the pull-back of Σ_{ABab} to the null surface is diagonalised into eigenspinors,*

$$\Sigma_{AB\underleftarrow{ab}} = \ell_{(A}\eta_{B)ab} .$$

Penrose notation:

- $A, B, C, \cdots = 0, 1$ transform under the fundamental representation of $SL(2, \mathbb{C})$.
- $A', B', C', \ldots$ transform under the complex conjugate representation.
- Indices raised and lowered with skew symmetric ϵ-spinor: $\ell_A = \epsilon_{BA}\ell^B$, $\ell^A = \epsilon^{AB}\ell_B$.

Intrinsic null geometry in terms of η_{Aab} and ℓ^A



- The p-form spinors (η_{Aab}, ℓ^A) determine entire intrinsic geometry of $\mathscr{N}$.
- Extrinsic geometry $(\sigma, \vartheta, \kappa)$ encoded into covariant exterior derivatives of (η_{Aab}, ℓ^A).
- There is now an additional $U(1)_{\mathbb{C}}$ gauge symmetry.
- The spinors are charged, but all vectors are $U(1)$ charge neutral (Penrose flag invisible).

	vectors and tensors	one spinorial object (η_{Aab}, ℓ^A)
null generator	$\ell^a : g_{ab}\ell^a\ell^b = 0$	$\ell^{AA'} = \mathrm{i}\ell^A\bar{\ell}^{A'}$
bivectors	$\Sigma_{\alpha\beta} = e_\alpha \wedge e_\beta$	$\Sigma_{AB\underleftarrow{ab}} = \ell_{(A}\eta_{B)ab}.$
scalar	$\boldsymbol{\varepsilon}_{ab} = 2\mathrm{i}\, m_{[a}\bar{m}_{b]}$	$\boldsymbol{\varepsilon}_{ab} = -\mathrm{i}\eta_{Aab}\ell^A$

- Metrical area

$$\mathrm{Area}_g[\mathscr{C}] = \int_{\mathscr{C}} \mathrm{d}s\,\mathrm{d}t \sqrt{\begin{pmatrix} g(\partial_s,\partial_s) & g(\partial_s,\partial_t) \\ g(\partial_t,\partial_s) & g(\partial_t,\partial_t). \end{pmatrix}}$$

- Oriented area flux

$$\mathrm{Area}_{\boldsymbol{\varepsilon}}[\mathscr{C}] = -\mathrm{i}\int_{\mathscr{C}} \eta_A \ell^A.$$

- Relative sign distinguishes ingoing from outgoing null boundaries.
- Analogous to the two natural volume elements on the manifold,

$$d^4x\sqrt{-\det g_{\mu\nu}} = \pm\frac{1}{4!}\epsilon_{\alpha\beta\mu\nu}e^\alpha \wedge e^\beta \wedge e^\mu \wedge e^\nu.$$

The boundary spinors enter the action through boundary terms.

- Tetradic Hilbert – Palatini action in the bulk,

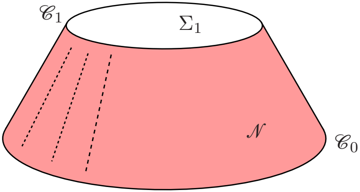
$$S_{\mathscr{M}}[A,e] = \frac{\mathrm{i}}{8\pi G}\int_{\mathscr{M}} \Sigma_{AB}\wedge F^{AB} + \mathrm{cc}.$$

- $SL(2,\mathbb{C})$-invariant boundary action,

$$S_{\mathscr{N}}[A|\eta,\ell] = \frac{\mathrm{i}}{8\pi G}\int_{\mathscr{N}} \eta_A \wedge D\ell^A + \mathrm{cc}.$$

- The variation of the action determines both the equations of motion and the symplectic potential.

$$\boxed{\delta S = \mathrm{EOM}\cdot\delta + \Theta_{\partial\mathscr{M}}(\delta).}$$



- $\Theta_{\partial\mathscr{M}} = \Theta_{\Sigma_1} + \Theta_{\Sigma_0} + \Theta_{\mathscr{N}}$.
- The symplectic potential acquires an additional corner term (arising from the boundary of the null boundary).

Symplectic structure along portion of the null surface:

$$\Theta_{\mathscr{N}} = \frac{\mathrm{i}}{8\pi G}\Big[\int_{\mathscr{N}}(D\ell^A)\wedge\delta\eta_A - (D\wedge\eta_A)\delta\ell^A\Big] + \mathrm{cc}.$$

- *Limit $\mathscr{N}\to\mathscr{I}^+$ returns the symplectic structure of the two radiative modes.*
- *Limit $\mathscr{N}\to\Delta_{\mathrm{IH}}$ returns the isolated horizon symplectic structure.*

Corner term in the symplectic potential

Covariant pre-symplectic potential for the partial Cauchy surfaces Σ

$$\Theta_\Sigma = \left[-\frac{\mathrm{i}}{8\pi G}\int_{\mathscr{C}} \eta_A \delta \ell^A + \frac{\mathrm{i}}{8\pi G}\int_\Sigma \Sigma_{AB} \wedge \delta A^{AB}\right] + \mathrm{cc}.$$

Poisson brackets at the two-dimensional corner

$$\big\{\eta_{Aab}(z), \ell^B(z')\big\}_{\mathscr{C}} = 8\pi\mathrm{i}G\delta^B_A \underset{\sim}{\epsilon}_{ab}\delta^{(2)}(z,z').$$

The area two-form is real only if a constraint is satisfied: vanishing of the $U(1)$ charge, (all observable are charge neutral)

$$L_\varphi[\mathscr{C}] = -\frac{1}{16\pi G}\int_{\mathscr{C}} \varphi\Big[\eta_A \ell^A + \mathrm{cc.}\Big] = 0, \quad \forall\varphi : \mathscr{C} \to [0, 2\pi].$$

Phase space, boundary observables

Quasi local observables

diffeomorphisms: $$J_\xi[\mathscr{C}] = \frac{\mathrm{i}}{8\pi G}\int_{\mathscr{C}}\Big[\eta_A \mathscr{L}_\xi \ell^A - \mathrm{cc.}\Big], \text{ for all } \xi^a\big|_{\mathscr{C}} \in T\mathscr{C}.$$

dilatations:* $$K_\lambda[\mathscr{C}] = -\frac{\mathrm{i}}{16\pi G}\int_{\mathscr{C}}\lambda\Big[\eta_A\ell^A - \mathrm{cc.}\Big] = \frac{1}{8\pi G}\int_{\mathscr{C}}\lambda\boldsymbol{\varepsilon}.$$

*S Carlip and C Teitelboim, The Off-shell black hole, Class. Quant. Grav. 12 (1995), `arXiv:gr-qc/9312002`.

Gauge symmetries (degenerate directions of $\Omega_\Sigma = \delta\Theta_\Sigma$)

- $U(1)$ flag rotations (all observables are charge neutral), $L_\varphi[\mathscr{C}] = 0$.

$$\eta_{Aab} \rightarrow \mathrm{e}^{-\frac{\mathrm{i}\varphi}{2}}\eta_{Aab}, \qquad \ell^A \rightarrow \mathrm{e}^{+\frac{\mathrm{i}\varphi}{2}}\ell^A.$$

- $SL(2,\mathbb{C})$ transformations of the bulk plus boundary fields.
- Bulk diffeomorphisms that vanish at the boundary.

Fock quantization of geometry

Inclusion of the Barbero – Immirzi parameter γ

The Barbero – Immirzi parameter $\gamma > 0$ is a coupling constant in front of the term $e_\alpha \wedge e_\beta \wedge F^{\alpha\beta}$, which can be added without changing the equations of motion. For $\gamma \neq 0$, we must modify then the boundary action as well.

- Bulk action

$$S_{\mathscr{M}}[A, e] = \frac{\mathrm{i}}{8\pi G}\Big[\int_{\mathscr{M}} \frac{\gamma + \mathrm{i}}{\gamma}\, \Sigma_{AB} \wedge F^{AB}\Big] + \mathrm{cc.}$$

- Boundary action for the null surface

$$S_{\mathscr{N}}[A|\eta, \ell] = \frac{\mathrm{i}}{8\pi G}\Big[\int_{\mathscr{N}} \frac{\gamma + \mathrm{i}}{\gamma}\, \eta_A \wedge D\ell^A\Big] + \mathrm{cc.}$$

- Canonical momentum (spinor-valued two-form on the boundary)

$$\boldsymbol{\pi}_A := \frac{\mathrm{i}}{8\pi G}\frac{\gamma + \mathrm{i}}{\gamma}\eta_A .$$

Reality conditions and Poisson brackets

- The Poisson brackets for the boundary variables are

$$\big\{\boldsymbol{\pi}_A(z), \ell_B(z')\big\} = \epsilon_{AB}\delta^{(2)}(z,z').$$

- Generator of complexified $U(1)_{\mathbb{C}}$ transformations

$$L = -\frac{1}{2\mathrm{i}}\boldsymbol{\pi}_A\ell^A + \mathrm{cc.} \quad \textit{(generator of U(1) transformations)},$$
$$K = -\frac{1}{2}\boldsymbol{\pi}_A\ell^A + \mathrm{cc.} \quad \textit{(dilatations of the null normal).}$$

- Cross-sectional area is neither L nor K, but

$$\boldsymbol{\varepsilon} = -8\pi G\frac{\gamma}{\gamma+\mathrm{i}}\boldsymbol{\pi}_A\ell^A.$$

- For the area to be real-valued, we have to satisfy reality conditions,

$$\boldsymbol{\varepsilon} = \bar{\boldsymbol{\varepsilon}} \Leftrightarrow \boxed{K - \gamma L = 0.}$$

- Poisson brackets in the continuum

$$\{\boldsymbol{\pi}_A(z), \ell^B(z')\} = \delta^B_A \delta^{(2)}(z, z').$$

- Strategy: Construct creation and annihilation operators and quantise them in the continuum
- This requires *two* additional structures:

$$\textit{Fiducial hermitian metric:} \quad \delta_{AA'} = \sigma_{AA'\alpha} n^\alpha,$$
$$\textit{Fiducial area element:} \quad d^2\Omega = \Omega^2(\vartheta, \varphi) \sin^2\vartheta \, \mathrm{d}\vartheta \wedge \mathrm{d}\varphi.$$

- Gravitational Landau operators (half densities)

$$a^A = \frac{1}{\sqrt{2}} \Big[\sqrt{d^2\Omega}\, \delta^{AA'} \bar{\ell}_{A'} - \frac{\mathrm{i}}{\sqrt{d^2\Omega}} \boldsymbol{\pi}^A \Big],$$
$$b^A = \frac{1}{\sqrt{2}} \Big[\sqrt{d^2\Omega}\, \ell^A + \frac{\mathrm{i}}{\sqrt{d^2\Omega}} \delta^{AA'} \bar{\boldsymbol{\pi}}_{A'} \Big].$$

- Poisson brackets

$$\{a^A(z), a^*_B(z')\} = \{b^A(z), b^*_B(z')\} = \mathrm{i}\delta^B_A \delta^{(2)}(z, z').$$

- Fock vacuum in the continuum

$$\forall z \in \mathscr{C} : a^A(z)\big|\{d^2\Omega, n_\alpha\}, 0\big\rangle = 0,$$
$$b^A(z)\big|\{d^2\Omega, n_\alpha\}, 0\big\rangle = 0.$$

- Imposition of the reality conditions:

$$\hat{L}(z) = \frac{1}{2}\big[a^\dagger_A(z)a^A(z) - b^\dagger_A(z)b^A(z)\big],$$
$$\hat{K}(z) = \frac{1}{2\mathrm{i}}\big[a_A(z)b^A(z) - \mathrm{hc.}\big],$$
$$\boxed{\big[\hat{K}(z) - \gamma\hat{L}(z)\big]\Psi_{\mathrm{phys}} = 0.}$$

- $\hat{K}$ is a squeeze operator, $\hat{L}$ plays the role of intrinsic spin.
- Physical states exhibit quantization of area

$$\widehat{\mathrm{Area}_\varepsilon[\mathscr{C}]}\Psi_{\mathrm{phys}} = 4\pi\gamma G\int_{\mathscr{C}}\big[a^\dagger_A a^A - b^\dagger_A b^A\big]\Psi_{\mathrm{phys}}.$$

- Possible measurement outcomes for cross-sectional area of $\mathscr{N}$

$$a_n = \frac{8\pi\gamma\,\hbar G}{c^3}n, \quad 2n \in \mathbb{Z}.$$

Conclusions

	spin network representation	boundary Fock representation
phase space	$\{E_i{}^a, A^j{}_b\} = 8\pi G\gamma\delta^j_i\delta^a_b\delta^{(3)}(\cdot,\cdot)$	$\{\boldsymbol{\pi}_A(z), \ell^B(z')\} = \delta^B_A\delta^{(2)}(z,z')$.
area	$\mathrm{Area}_g[\mathscr{C}] = \int_{\mathscr{C}} \sqrt{E_i E^i}$	$\mathrm{Area}_{\boldsymbol{\varepsilon}}[\mathscr{C}] = -8\pi G \frac{\gamma}{\gamma+\mathrm{i}} \int_{\mathscr{C}} \boldsymbol{\pi}_A \ell^A$
eigenvalues	$A_j = 8\pi G\,\gamma\sqrt{j(j+1)}, \quad 2j \in \mathbb{N}_0$	$a_j = 8\pi G\,\gamma\, j, \quad 2j \in \mathbb{Z}$

Where does the quantisation of area come from?

- Adding $\frac{1}{\gamma}\int_{\mathscr{M}} e_\alpha \wedge e_\beta \wedge F^{\alpha\beta}$ to the action does not change the EOMs.
- Introducing γ deforms the reality conditions: they contain now also the generator of $U(1)$ rotations.
- Quantization of area follows then from quantization of the $U(1)$ generators.

Summary

- We started with a heuristic argument: In LQG, the quantum states of geometry are built from gravitational Wilson lines for the spin connection. If these Wilson lines hit a boundary, they excite a surface charge, namely a spinor.
- We then found the classical interpretation for these surface spinors: The LQG boundary spinors appear already at the classical level as gravitational edge modes in the Hamiltonian formalism *in domains bounded by null surfaces*. The simplest quasi-local observables that can be constructed from these surface spinors are: (i) tangential diffeomorphisms that preserve the corners and (ii) dilatations of the null normal.
- Quantisation of area in conventional Fock space: Finally, we quantised the surface spinors using a conventional Fock representation (keeping the theory in the bulk classical). Upon introducing the Immirzi parameter, we reproduced the LQG quantisation of area without ever introducing spin networks or discretizations of space.